Math 122 / Problem Set 8

Written problems due Monday, November 14

Monday, November 7

1. Let p, q be permutations. Prove that the products pq and qp have cycles of equal sizes.

2. Show how to determine whether a permutation is odd or even when it is written as a product of cycles.

3. **(a)** Prove or disprove: The order of a permutation is the least common multiple of the orders of the cycles in its disjoint cycle decomposition.

(b) Does the symmetric group S_7 contain an element of order 5? of order 10? of order 15?

(c) What is the largest possible order of an element of S_7?

4. Find all subgroups of order four of the symmetric group S_4. Which are normal?

5. Compute the number of permutations in S_n which do not leave any index fixed.

Reading: None

Wednesday, November 9

6. Prove that there is no sequence of moves on a Rubik's Cube to switch two edge pieces while leaving the rest unchanged.

7. Show that the commutator $[g, h]$ of $g, h \in S_n$ is trivial if g and h have disjoint support. Is the converse true?

8. Let $H \subset G$ be the smallest subgroup containing all commutators $[g, g']$ of elements of G. Show that H is normal in G and that the quotient group G/H is abelian.

Reading: None